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The energy transfer from submarine seismic waves to the ocean

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Similar formulas are obtained for the other combinations of U and W and we end up with

$$E(k,t)=E(k,o)\left\{1-\tfrac{1}{2}\frac{\lambda_2^2 k_n}{\lambda_1^2 h(k-k_n)^2+\lambda_2^2 k_n}(1-\cos k_n(c^{(n+1)}-c^{(n)})t)\right\} \tag{9.6}$$

where $E(k,t)$ denotes the complete energy density in the bottom layer for wave number k and time t.

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EWING, M., W. JARDETZKY and F. PRESS, 1957: *Elastic Waves in Layered Media.* New York.

PALM, E., 1953: On the formation of surface waves in a fluid flowing over a corrugated bed and on the development of mountain waves. *Astroph. Norv.* **5**, No. 3.

9. The total energy in the bottom layer. ξ and ζ denote the displacement in the x- and z-direction, respectively. We then have

$$\xi_t=\phi_{xt}-\psi_{zt}, \qquad \zeta_t=\phi_{zt}+\psi_{xt}. \tag{9.1}$$

Applying the theory of Fourier Transform we write

$$\begin{aligned}\xi_t&=\mathrm{Re}\int_0^\infty (U_1+U_2)\exp(ikx)dk\\ \zeta_t&=\mathrm{Re}\int_0^\infty (W_1+W_2)\exp(ikx)dk\end{aligned} \tag{9.2}$$

where, according to (5.15)

$$\begin{aligned}U_1&=kc_Ra(k)\sum_n\left\{\frac{(1+m_2^2)\exp(km_1z)}{m_1(1-m_2^2)}\frac{Q\sin km_0h}{\partial D/\partial c}\right\}_n\frac{\exp(-ikc^{(n)}t)}{c^{(n)}-c_R}\\ U_2&=-2kc_Ra(k)\sum_n\left\{\frac{m_2\exp(km_2z)}{1-m_2^2}\frac{Q\sin km_0h}{\partial D/\partial c}\right\}_n\frac{\exp(-ikc^{(n)}t)}{c^{(n)}-c_R}\\ W_1&=-ikc_Ra(k)\sum_n\left\{\frac{(1+m_2^2)\exp(km_1z)}{1-m_2^2}\frac{Q\sin km_0h}{\partial D/\partial c}\right\}_n\frac{\exp(-ikc^{(n)}t)}{c^{(n)}-c_R}\\ W_2&=2ikc_Ra(k)\sum_n\left\{\frac{\exp(km_2z)}{1-m_2^2}\frac{Q\sin km_0h}{\partial D/\partial c}\right\}_n\frac{\exp(-ikc^{(n)}t)}{c^{(n)}-c_R}\end{aligned} \tag{9.3}$$

The total energy in the lowest layer is twice the kinetic energy and is therefore given by

$$\begin{aligned}\int_{-\infty}^{+\infty}dx\int_{-\infty}^{0}(\xi_t^2+\zeta_t^2)dz&=\pi\int_0^\infty\int_{-\infty}^0(U_1U_1^*+U_1U_2^*+U_1^*U_2+U_2U_2^*)dkdz\\ &+\pi\int_0^\infty\int_{-\infty}^0(W_1W_1^*+W_1W_2^*+W_1^*W_2+W_2W_2^*)dkdz\end{aligned} \tag{9.4}$$

Applying an expansion about $k=k_n$ and $c=c_R$, as above, utilizing (8.6), (8.10) and (8.11) we obtain

$$\begin{aligned}\int_{-\infty}^{0}U_1U_1^*dz&=\tfrac{1}{2}k_nc_R^2|a(k)|^2\left\{\frac{(1+m_2^2)^2}{m_1^3(1-m_2^2)^2}\right\}_{c=c_R}\\ &\cdot\left(1-\tfrac{1}{2}\frac{\lambda_2^2k_n}{\lambda_1^2h(k-k_n)^2+\lambda_2^2k_n}(1-\cos k_n(c^{(n+1)}-c^{(n)})t)\right)\end{aligned} \tag{9.5}$$

We then have

$$D=-\frac{k_nhc_RR'(c_R)}{m_0c_0^2}\sin km_0h\left((c-c_R)^2+\frac{m_0^2c_0^2}{k_nc_R}(k-k_n)(c-c_R)-\frac{m_0c_0^2Q(c_R)}{k_nhc_RR'(c_R)}\right) \tag{8.6}$$

$$=-\frac{k_nhc_RR'(c_R)\sin km_0h}{m_0c_0^2}(c-c^{(n)})(c-c^{(n+1)}).$$

Furthermore,

$$\left(\frac{cR\sin km_0h}{m_0\partial D/\partial c}\right)_n\left(\frac{R\cos km_0h}{\partial D/\partial c}\right)_{n+1}= \tag{8.7}$$

$$\frac{c_0^2}{k_nhc_R}\frac{c_R(c^{(n)}-c_R)(c^{(n+1)}-c_R)\left(c^{(n+1)}-c_R+\dfrac{m_0^2c_0^2}{k_nc_R}(k-k_n)\right)}{(c^{(n+1)}-c^{(n)})^2}$$

$$\left(\frac{cR\sin km_0h}{m_0\partial D/\partial c}\right)_{n+1}\left(\frac{R\cos km_0h}{\partial D/\partial c}\right)_n= \tag{8.8}$$

$$\frac{c_0^2}{k_nhc_R}\frac{c_R(c^{(n+1)}-c_R)(c^{(n)}-c_R)\left(c^{(n)}-c_R+\dfrac{m_0^2c_0^2}{k_nc_R}(k-k_n)\right)}{(c^{(n+1)}-c^{(n)})^2}$$

We define A_n by

$$A_n=\left(\frac{cR\sin km_0h}{m_0\partial D/\partial c}\right)_n\left(\frac{R\cos km_0h}{\partial D/\partial c}\right)_{n+1}-\left(\frac{cR\sin km_0h}{m_0\partial D/\partial c}\right)_{n+1}\left(\frac{R\cos km_0h}{\partial D/\partial c}\right)_n \tag{8.9}$$

which with the above approximations takes the form

$$A_n=\frac{c_0^2}{k_nh}\frac{(c^{(n+1)}-c_R)(c^{(n)}-c_R)}{c^{(n+1)}-c^{(n)}}.$$

From (8.6) we find

$$c^{(n)}-c_R=c_R[-\lambda_1/k_n(k-k_n)-(\lambda_1^2/k_n^2(k-k_n)^2+\lambda_2^2/k_n^2h^2)^{\frac{1}{2}}] \tag{8.10}$$

$$c^{(n+1)}-c_R=c_R[-\lambda_1/k_n(k-k_n)+(\lambda_1^2/k_n^2(k-k_n)^2+\lambda_2^2/k_n^2h^2)^{\frac{1}{2}}] \tag{8.11}$$

where λ_1 and λ_2 are defined by (5.13) and (5.14). Inserting these expressions in (8.9) and (5.11) and applying the relation

$$\lambda_2^2=\frac{1}{2B}\frac{\rho_0c_0^2}{\rho_0c_R^2} \tag{8.12}$$

we end up with (5.12).

where

$$\beta_1^2 = 1 - c_R^2/c_1^2, \qquad \beta_2^2 = 1 - c_R^2/c_2^2. \tag{7.3}$$

$A_1(k)$ and $A_2(k)$ are found from the boundary conditions ($\sigma_z = \tau_{xz} = 0$ and ζ given by (3.1) for $z=0$) to be

$$A_1(k) = -\frac{(1+\beta_2^2)a(k)}{\beta_1(1-\beta_2^2)k}$$

(7.4)

$$A_2(k) = -\frac{2ia(k)}{(1-\beta_2^2)k}$$

c_R are determined by

$$(1+\beta_2^2)^2 - 4\beta_1\beta_2 = 0 \tag{7.5}$$

Inserting (7.2) in (7.1) and applying Parseval's theorem, it is obtained that

$$\varepsilon = \rho\pi c_R^2 B \int_0^\infty k|a(k)|^2 dk \tag{7.6}$$

where

$$B = \frac{(1+\beta_1^2)(1+\beta_2^2)^2}{2\beta_1^3(1-\beta_2^2)^2} - \frac{4(1+\beta_2^2)}{\beta_1(1-\beta_2^2)^2} + \frac{2(1+\beta_2^2)}{\beta_2(1-\beta_2^2)^2}$$

Since c_R/c_1 and c_R/c_2 depend only on the Poisson ratio ν, B is a function of ν only. For $\nu = 0{,}25$, we find $B = \sqrt{6\sqrt{3}} \approx 3{,}2237$.

8. Approximate evaluation of the energy flux when $k \approx k_n$ and $c \approx c_R$ and ρ_0/ρ is small.

The determinant D may be written

$$D = R(c)\cos km_0h + Q(c)\sin km_0h \tag{8.1}$$

Now,

$$R(c) \approx R'(c_R)(c-c_R) \tag{8.2}$$

$$Q(c) \approx Q(c_R) \tag{8.3}$$

$$\cos km_0h \approx \frac{\partial}{\partial c}(\cos km_0h)_{\substack{c=c_R\\ kn=k}}(c-c_R) + \frac{\partial}{\partial k}(\cos km_0h)_{\substack{c=c_R\\ k=k_n}}(k-k_n) \tag{8.4}$$

$$\sin km_0h \approx (\sin km_0h)_{\substack{c=c_R\\ k=k_n}} \tag{8.5}$$

For simplicity we neglect the subscripts $c=c_R$, $k=k_n$ when no misunderstanding is possible.

When the seismic wave has passed the ocean region, the energy conveyed to the ocean is lost for the seismic wave and therefore represents a kind of dissipation. A Fourier analysis of the energy spectrum of such a wave therefore should reveal that no, or only little, energy has been lost in the part of the spectrum corresponding to periods larger than T_0. For the periods about T_0, T_1, T_2, ... (the periods corresponding to $k=k_0$, k_1, ...), however, the energy spectrum should possess typical minima. We do not know if any such tendencies have been observed. However, according to EWING, JARDETZKY and PRESS (1957), observations in the Pacific region show that waves with periods less than 12 sec (and larger than 1 sec, to be exact) suffer a great attenuation in typical ocean areas. Thus, good agreement exists between theory and observations, the theory above implying that waves with periods larger than 11,8 sec (when $h=5000$ m) do not lose any energy to the ocean. There is, however, also a discrepancy, since in the present theory the attenuation of the energy will be very unequally distributed in the energy spectrum, having distinct maxima for $k=k_0, k_1, k_2, \ldots$. Besides, the computed maximum energy loss never exceeds 50%. This energy loss would likely have been somewhat larger if friction in the fluid layer had been taken into account. It seems reasonable that a small friction in the fluid layer will not change the main kinematical picture. The dissipation due to such a friction is proportional to the square of the vorticity and therefore proportional to k^2 times the kinetic energy. We therefore get most dissipation for k-values corresponding to large energy — which again corresponds to large energy flux at the interface. To compensate for this dissipation the energy flux increases and therefore the introduction of friction results in the largest increase of the energy transport where it already has its largest value. In other words, in the part of the energy spectrum where the transport is small, it should still remain small; whereas, in the part of the spectrum where it is large, it should further increase.

We have assumed above that the velocity of sound is constant with height. Taking into account the observed variation of the velocity of sound, our results will be somewhat modified for smaller periods.

APPENDIX

7. The total energy in the Rayleigh wave. The total energy, ε, in a Rayleigh wave is twice the kinetic energy. Therefore

$$\varepsilon=\rho\int_{-\infty}^{0}dz\int_{-\infty}^{+\infty}(\xi_t^2+\zeta_t^2)dx. \tag{7.1}$$

It is easily seen that ϕ and ψ have the form

$$\begin{aligned}\phi&=\mathrm{Re}\int_0^\infty A_1(k)\exp\{k\beta_1 z+ik(x-c_Rt)\}dk\\ \psi&=\mathrm{Re}\int_0^\infty A_2(k)\exp\{k\beta_2 z+ik(x-c_Rt)\}dk\end{aligned} \tag{7.2}$$

It is noticed that for large values of time the energy spectrum for the bottom layer is reduced to half of its initial value for $k=k_n$, in agreement with the result found by considering the energy flux at the interface. In Fig. 5 the energy spectrum is shown for the bottom layer at large values of time. The correspondence between Figs. 4 and 5 are given by the energy equation

$$E(k,t)+\pi\text{Re}\int_0^t PW^* dt = E(k,o)\,. \tag{5.18}$$

In the procedure above, we have integrated over a band of k-values and in equations (5.10) and (5.16) we have neglected the trigonometric terms, assuming τ (or t) to be sufficiently large. How large must τ (or t) be for this approximation to be valid? There is no unique answer to this question, the time depending on the chosen width of the band. This is again a question about the dispersion power of the instruments.

6. Discussion of the results. Conclusion. We have above considered the energy transport connected with a seismic wave travelling across an ocean basin. Our model is taken to be a two-layer system, a bottom layer which is a perfect elastic medium and an uppermost layer which is a perfect fluid. At $t=0$, the fluid is assumed to be at rest, whereas the bottom layer possesses a motion corresponding to a Rayleigh wave (these initial conditions are thought to be the proper ones for the case of a seismic wave travelling across a continent at $t<0$ and across an ocean for $t>0$.) The energy which, after a sufficient large span of time, has been conveyed to the fluid, is then computed. It is found that waves with wave numbers less than a certain value, or, equivalently, with periods larger than a certain value, lose only a very small fraction (with our approximation nothing) of their energy to the fluid. More precisely, inserting characteristic values of c_R and c_0 and putting $h=5000$ m (a typical value for the Pacific) we find that no energy is transferred to the fluid layer for wave numbers less than 0,59/h or, equivalently, periods larger than 11,8 sec. For this wave number, the energy transfer has an abrupt increase, and then very quickly obtains its first maximum value for $k=k_0$, (see Fig. 4). The energy transfer then decreases rapidly until it again has an abrupt increase for a value of k a little less than k_1. For $k=k_1$, it has its second maximum value, and so on. It should be noted that the values of k for which the energy transfer has its maxima, $k=k_0, k_1, \ldots$, are the k-values for which resonance occurs when the waves in the bottom layer are assumed to move independently of the occurrence of the uppermost layer, i.e. as a Rayleigh wave.

The reason for the discontinuous changes in the energy transport at the interface is that all $c^{(n)}(k)$ except $c^{(0)}$ do not exist for all k-values. In order to have any energy transfer for a k-value, at least two $c^{(n)}(k)$-curves must exist for this value of k. Where only one $c^{(n)}(k)$ occurs, no energy transfer takes place. The most important feature as to the energy transport is, however, not these discontinuities and the existence of only one $c^{(n)}(k)$-curve in a certain k-region, but the result that the energy transport is concentrated about $k=k_0, k_1, \ldots$ for the characteristic values of the parameters applied.

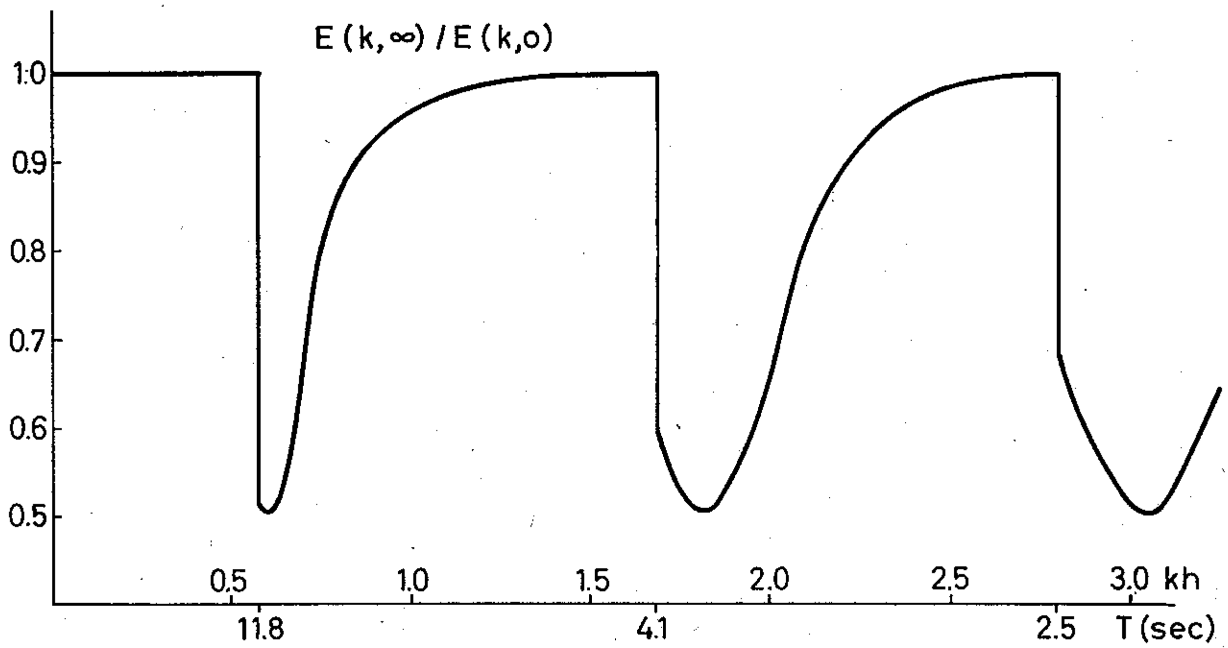

Figure 5. The energy in the bottom layer for large values of time as function of k and the period T when $\varrho_0/\varrho = 1/3$, $\nu = 1/4$, $c_0 = 1500$ m/sec, $c_2 = 4500$ m/sec and $h = 5000$ m.

The energy flux at the interface may also be found by considering the energy in the bottom layer at various values of time. The displacement potentials ϕ and ψ are given by (4.5) and (4.6). Retaining only the contributions from the poles and considering only the waves advancing in positive x-direction, in agreement with the procedure above, the Fourier transforms of ϕ and ψ are given by

$$\Phi = \frac{c_R a(k)}{k} \sum_n \left\{ \frac{(1+m_2^2)\exp(km_1 z)}{m_1(1-m_2^2)} \frac{Q \sin km_0 h}{c\partial D/\partial c} \right\}_n \frac{\exp(-ikc^{(n)}t)}{c^{(n)}-c_R} \tag{5.15}$$

$$\Psi = \frac{2ic_R a(k)}{k} \sum_n \left\{ \frac{\exp(km_2 z)}{1-m_2^2} \frac{Q \sin km_0 h}{c\partial D/\partial c} \right\}_n \frac{\exp(-ikc^{(n)}t)}{c^{(n)}-c_R}$$

In the Appendix, section 9, the energy spectrum for the bottom layer, $E(k, t)$, is evaluated for various values of time. For k-values in the vicinity of k_n, it is found that $E(k, t)$ is approximately

$$E(k,t) = E(k,o)\left\{1 - \tfrac{1}{2}\frac{\lambda_2^2 k_n}{\lambda_1^2 h(k-k_n)^2 + \lambda_2^2 k_n}(1-\cos k_n(c^{(n+1)}-c^{(n)})t)\right\} \tag{5.16}$$

where $E(k, o)$ is the energy spectrum for the Rayleigh wave, i.e. $E(k, o) = E(k)$ defined by (3.16). Integration over a small band of k-values makes the periodic term in (5.16) tend towards zero when $t \to \infty$ such that for large values of time

$$E(k,t) = E(k,o)\left(1 - \tfrac{1}{2}\frac{\lambda_2^2 k_n}{\lambda_1^2 h(k-k_n)^2 + \lambda_2^2 k_n}\right) \tag{5.17}$$

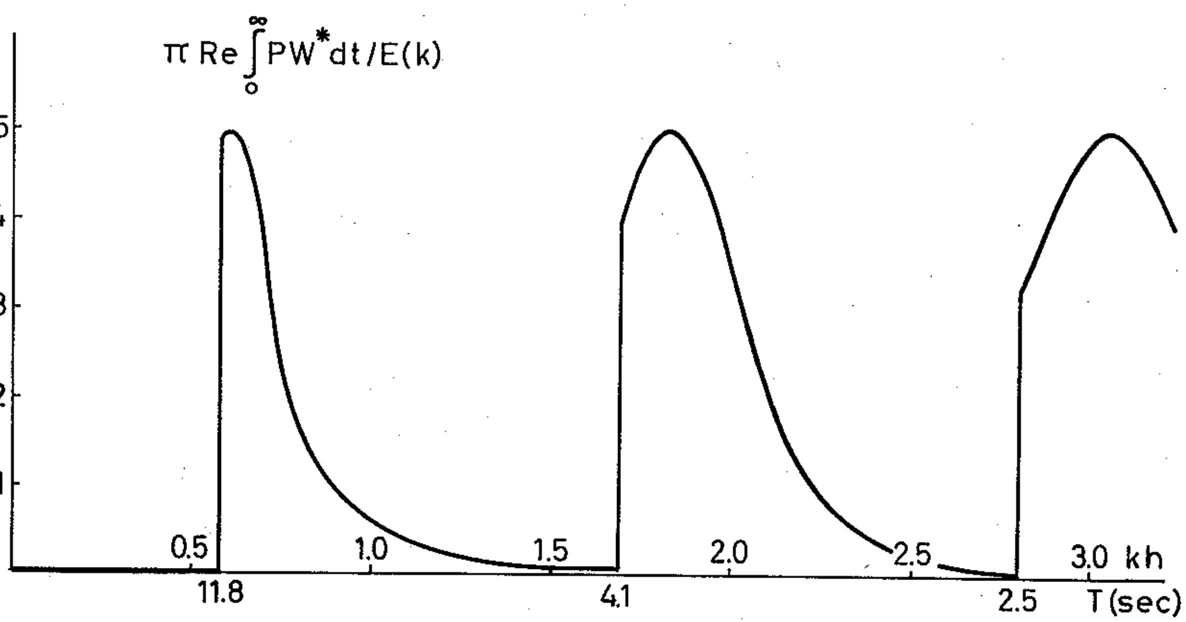

Figure 4. The time-integrated energy transport at the interface as function of k. The curves show this energy transport in proportion to the initial energy in the bottom layer. Moreover the figure displays this energy transport as function of the period T when $\varrho_0/\varrho = 1/3$, $\nu = 1/4$, $c_0 = 1500$ m/sec, $c_2 = 4500$ m/sec and $h = 5000$ m.

somewhat less. In Fig. 4 is shown the energy transport integrated over time when the curves are obtained by using the approximate form (5.12) in the vicinity of $k = k_n$ and evaluating the right hand side of (5.11) for some few values of k when the approximation is not supposed to be good.

It is seen from Fig. 4 that the energy transport integrated over time is a discontinuous function. For values of k less than 0,59/h (corresponding to periods larger than 11,8 sec) there is no energy transport. For $k = 0{,}59/\mathrm{h}$ an abrupt change in the energy transport occurs, the energy transport obtaining nearly its maximum value. For higher values of k the energy transport decreases rapidly until it again obtains an abrupt change for $k = 1{,}70/\mathrm{h}$ (corresponding to $T = 4{,}1$ sec). This picture will be repeated over and over again for increasing values of k. The reason for this somewhat curious behaviour of the energy transport is due to the fact that $c^{(1)}$, $c^{(2)}$, ... do not exist for all values of k. For k less than 0,59/h, $c^{(1)}$ does not exist, and $c^{(0)}$ alone is not able to perform any energy transport (since the streamline pattern does not have any tilt), and therefore RePW^* is zero for these values of k. For larger values of k, $c^{(0)}$ and $c^{(1)}$ together give positive energy transport which rapidly decreases for increasing values of $k - k_0$. For k larger than 1,70/h $c^{(2)}$ exists and the main energy transport is due to the waves $c^{(1)}$ and $c^{(2)}$ in the interval about $k = k_1$. For still higher values of k this energy transport decays, and the energy transport is mainly due to $c^{(2)}$ and $c^{(3)}$ and so on. It should be noticed that discontinuities of this kind in the energy transport always will occur when there are several waves, some of which do not exist for all k-values.

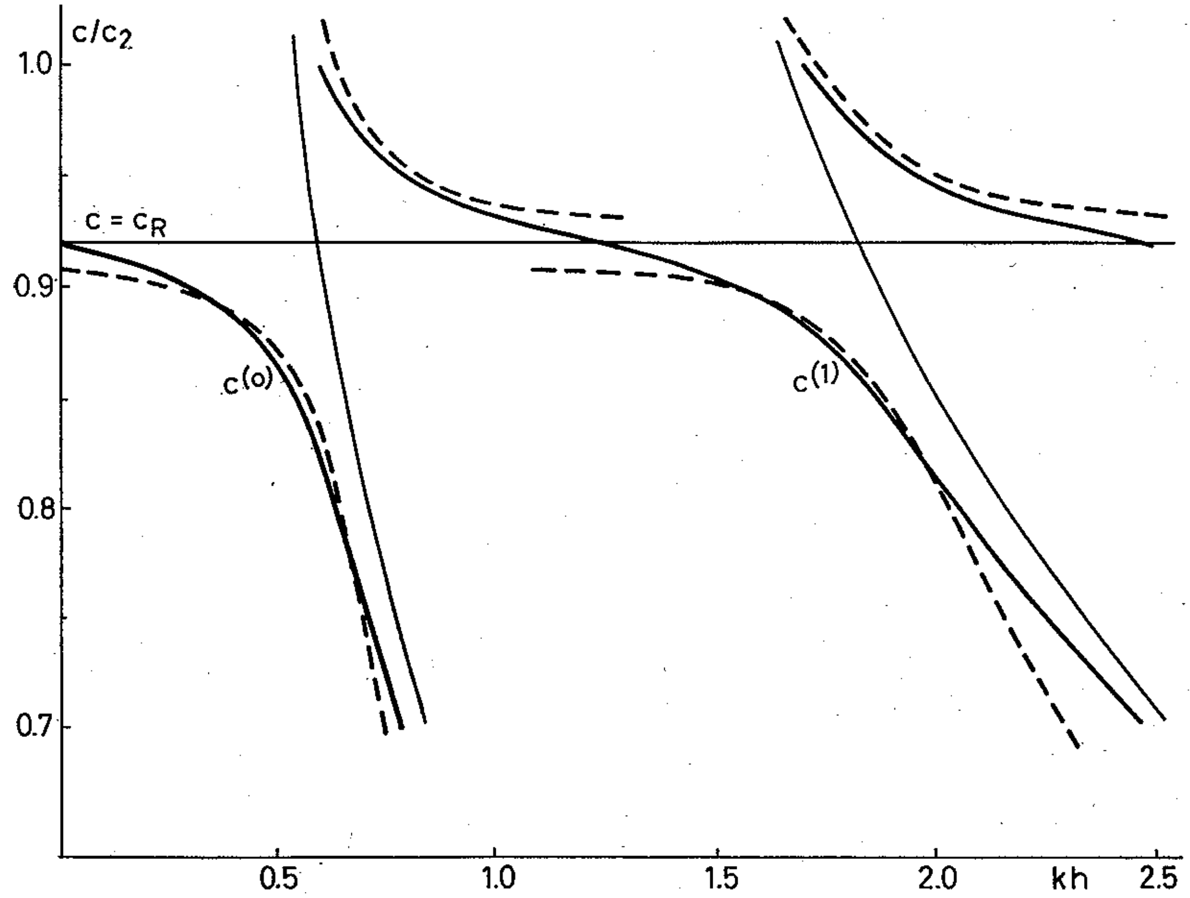

Figure 3 displays the approximate dispersion curves (dashed lines) compared with the exact dispersion curves (solid lines).

putting $\rho_0/\rho = 1/3$. The approximation is seen to be rather good, and we shall apply these expressions for $c^{(n)}(k)$ in the neighbourhood of $k = k_n$.

According to the Appendix, section 8, the energy transport, valid in the vicinity of $k = k_n$, is then given by

(5.12) $$\pi \mathrm{Re} \int_0^\tau PW^* dt = \tfrac{1}{2} E(k) \frac{\lambda_2^2 k_n}{\lambda_1^2 h (k-k_n)^2 + \lambda_2^2 k_n}$$

where $E(k)$ is given by (3.16), and

(5.13) $$\lambda_1 = \tfrac{1}{2}(1 - c_0^2/c_R^2)$$

(5.14) $$\lambda_2^2 = (c_0/c_R) Q(c_R)(1 - c_0^2/c_R^2)^{\frac{1}{2}} / c_R R'(c_R).$$

It is seen that (5.12) has its maximum values for $k = k_n$, and for these k-values the energy flux integrated over time is equal to $\tfrac{1}{2}E(k_n)$. Since $E(k)$ is the energy spectrum at $t = 0$ in the bottom layer, this means that *the bottom layer has lost half of its energy at* $k = k_n$ *when* t *is sufficiently large.* For other values of k the energy transport will be are by far the most important ones. These terms have the smallest denominators being the product of two differences. Furthermore, these terms have a slower variation with time than the other terms, the frequency also being a difference. This last feature becomes important when the energy transport is integrated over time. Such an integration shows that terms of type (5.7) will have denominators being a product of three differences whereas the other terms will have denominators being a product of one difference and two sums. Furthermore, it is seen that, when integrating over k, the most important contributions are due to values of k and c for which $k \approx k_n$ and $c \approx c_R$. We may therefore with a good approximation retain only terms of type (5.7) and when integrating over k, in the neighbourhood of k_n take into account only the contributions from $c^{(n)}$ and $c^{(n+1)}$. Equation (5.6) then takes the form

$$\text{Re}PW^* = \rho_0 c_R^2 k^2 |a(k)|^2 A_n \frac{\sin k(c^{(n+1)} - c^{(n)})t}{(c^{(n)} - c_R)(c^{(n+1)} - c_R)} \tag{5.8}$$

where

$$A_n = \left(\frac{cR \sin km_0 h}{m_0 \partial D/\partial c}\right)_n \left(\frac{R \cos km_0 h}{\partial D/\partial c}\right)_{n+1} - \left(\frac{cR \sin km_0 h}{m_0 \partial D/\partial c}\right)_{n+1} \left(\frac{R \cos km_0 h}{\partial D/\partial c}\right)_n . \tag{5.9}$$

An inspection of Fig. 2 reveals that the error introduced by applying (5.8) instead of (5.6) is less than 1% in the immediate neighbourhood of $k = k_n$. For k-values right in between k_n and k_{n+1} the error will be somewhat larger. However, for these k-values the energy transport is very small. It is seen that using (5.8) instead of (5.6) corresponds to neglecting waves advancing in the negative x-direction in the solution (4.11).

The integrated energy flux from $t = 0$ to $t = \tau$ (τ will be assumed large), is then given by

$$\pi \text{Re} \int_0^\tau PW^* dt = \frac{\pi \rho_0 c_R^2 k |a(k)|^2 A_n (1 - \cos k(c^{(n+1)} - c^{(n)})\tau)}{(c^{(n)} - c_R)(c^{(n+1)} - c_R)(c^{(n+1)} - c^{(n)})} \tag{5.10}$$

Integration over a small band of k-values makes the periodic term in (5.10) tend towards zero when $\tau \to \infty$. This term will therefore be cancelled. We thus have, approximately.

$$\pi \text{Re} \int_0^\tau PW^* dt = \frac{\pi \rho_0 c_R^2 k |a(k)|^2 A_n}{(c^{(n)} - c_R)(c^{(n+1)} - c_R)(c^{(n+1)} - c^{(n)})} \tag{5.11}$$

which gives the energy transport as function of k. In principle we know this function by using $c^{(n)}$ as a function of k as found from Fig. 2. A_n is, however, a somewhat complicated function of $c^{(n)}$ and to obtain an analytical expression for $c^{(n)}(k)$, we develope D in a series about $k = k_n$ and $c = c_R$, valid for small values of ρ_0/ρ (see Appendix, section 8). In Fig. 3 is shown the approximate values of $c^{(n)}(k)$ obtained by this procedure (dashed lines) compared to the exact ones. These approximate curves are found by retaining only the leading terms in the development of D and

By means of (4.11) we then obtain

$$(5.3)\quad \begin{aligned} W=&-\sum_n\left\{\frac{ika(k)\exp(-ikct)c_R R\cos km_0h}{(c-c_R)\partial D/\partial c}\right\}_n \\ &-\sum_n\left\{\frac{ika(k)\exp(ikct)c_R R\cos km_0h}{(c+c_R)\partial D/\partial c}\right\}_n \end{aligned}$$

$$(5.4)\quad \begin{aligned} P=&-\sum_n\left\{\frac{\rho_0 ka(k)\exp(-ikct)c_R cR\sin km_0h}{m_0(c-c_R)\partial D/\partial c}\right\}_n \\ &+\sum_n\left\{\frac{\rho_0 ka(k)\exp(ikct)c_R cR\sin km_0h}{m_0(c+c_R)\partial D/\partial c}\right\}_n \end{aligned}$$

Furthermore, Parseval's theorem may be written

$$(5.5)\quad \int_{-\infty}^{+\infty} p(o)w(o)dx=\pi\mathrm{Re}\int_0^{\infty} PW^*dk$$

where an asterisk denotes the complex conjugate. By means of (5.3) and (5.4) we obtain

$$(5.6)\quad \begin{aligned} \mathrm{Re}PW^*=&-\rho_0c_R^2k^2|a(k)|^2\left\{\sum_n\left(\frac{cR\cos km_0h\sin km_0h}{m_0\partial D/\partial c}\right)_n^2\cdot\frac{2\sin 2kc^{(n)}t}{c^{(n)2}-c_R^2}\right. \\ &+\sum_{n\neq m}\left(\frac{cR\sin km_0h}{m_0\partial D/\partial c}\right)_n\left(\frac{R\cos km_0h}{\partial D/\partial c}\right)_m\left[\frac{\sin k(c^{(n)}-c^{(m)})t}{(c^{(n)}-c_R)(c^{(m)}-c_R)}\right. \\ &\left.\left.+\frac{\sin k(c^{(n)}+c^{(m)})t}{(c^{(n)}-c_R)(c^{(m)}+c_R)}+\frac{\sin k(c^{(n)}+c^{(m)})t}{(c^{(n)}+c_R)(c^{(m)}-c_R)}+\frac{\sin k(c^{(n)}-c^{(m)})t}{(c^{(n)}+c_R)(c^{(m)}+c_R)}\right]\right\} \end{aligned}$$

where subscripts n and m denote that c is given the values $c^{(n)}$ and $c^{(m)}$ respectively.

The motion is, according to (4.11) (or 5.3) and (5.4)), composed of two different wave systems, one advancing in the positive x-direction, the other advancing in the negative x-direction. The initial conditions have been chosen so that they correspond to a Rayleigh wave advancing in positive x-direction at $t=o$. The part of (4.11) advancing in negative x-direction therefore corresponds to a system of reflection waves which have been created since the initial conditions are not the proper ones for giving waves advancing only in one direction in the two-layer model. It seems reasonable that this system is not very important with respect to the energy transport, which may easily be demonstrated.

A discussion of the various terms in (5.6) combined with an inspection of Fig. 2 reveals that terms of the type

$$(5.7)\quad \frac{\sin k(c^{(n)}-c^{(m)})t}{(c^{(n)}-c_R)(c^{(m)}-c_R)}$$

which is the secular equation for the system. It may be shown that this equation has for fixed values of k a finite number of solutions, $c=\pm c^{(n)}$. Applying Cauchy's theorem the contributions from the branch points are usually neglected compared to the contributions from the poles, which is a good approximation for moderate values of time. With this approximation, we find

$$Z=\sum_n\left\{\frac{a(k)\exp(ikct)c_R R\cos km_0(h-z)}{c(c-c_R)\partial D/\partial c}\right\}_n \tag{4.11}$$
$$-\sum_n\left\{\frac{a(k)\exp(ikct)c_R R\cos km_0(h-z)}{c(c+c_R)\partial D/\partial c}\right\}_n$$

where the sum is taken over all positive $c^{(n)}$ determined from (4.10) and subscript n indicates that c is given the value $c^{(n)}$. It should be noted that neglecting the branch points corresponds to considering only a slightly different initial situation than originally specified.

In Fig. 2 is shown graphically the variation of $c^{(n)}$ as a function of k, found by solving (4.10) numerically (solid lines), for the case of $\rho_0/\rho=1/3$ and for given values of the other parameters. The system of dashed lines corresponds to the solutions of the equations $R=0$ (i.e. $c=c_R$) and $\cos km_0 h=0$. In the case of no coupling, (4.11) has poles corresponding to the points of intersection of these lines. In the actual case, no resonance occurs, and Z is therefore non-singular for all k-values (the apparent singularity for $c=c_R$ is really no singularity since then R is zero). However, in the case of weak coupling (i.e. for ρ_0/ρ relatively small), the denominator is small for values of k and c in the neighbourhood of the points of intersection. Therefore, when integrating over all k-values, the most important contributions come from such values of c and k. This will be utilized in the following section.

5. The energy flux. We shall now compute the flux of energy from the bottom layer to the uppermost layer in the case of weak coupling. This flux may be written as $\int_{-\infty}^{+\infty} p(o)w(o)dx$ where $p(o)$ is the pressure and $w(o)$ the vertical velocity at the interface. Writing

$$p=\mathrm{Re}\int_0^\infty P(k,z,t)\exp(ikx)dk \tag{5.1}$$
$$w=\mathrm{Re}\int_0^\infty W(k,z,t)\exp(ikx)dk$$

we find that

$$W=Z_t \tag{5.2}$$
$$P_z=-\rho_0 Z_{tt}.$$

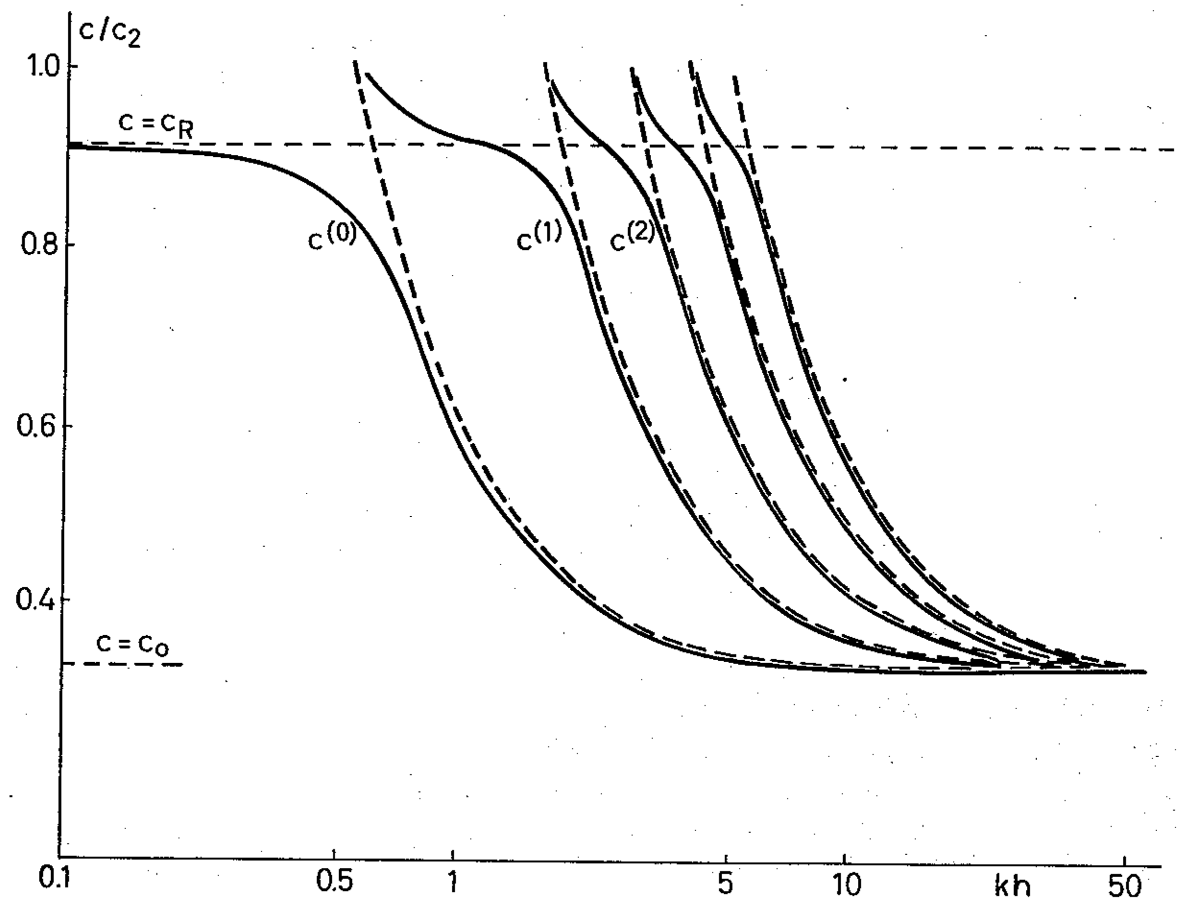

Figure 2. The solid lines represent the dispersion curves in the considered two-layer model when $\varrho_0/\varrho = 1/3$, $\nu = 1/4$, $c_0 = 1500$ m/sec and $c_2 = 4500$ m/sec. The dashed horizontal line corresponds to $c = c_R$. The other dashed curves show c as function of k as found from $\cos km_0 h = 0$.

Let us for the moment confine ourselves to the motion in the uppermost layer. The vertical displacement ζ may be written

$$\zeta = \mathrm{Re} \int_0^\infty Z(k, z, t) \exp(ikx) dk \tag{4.8}$$

From the relation $\zeta_t = \chi_z$, we easily find the Laplace transform of Z. By means of the inversion formula for the Laplace transform we then have

$$Z = -\frac{kc_R a(k)}{2\pi} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\exp(st) R \cos km_0(h-z)}{s(s+ikc_R)D} ds. \tag{4.9}$$

The integrand has branch points for $m_1 = 0$ and $m_2 = 0$, i.e. for $s = \pm ikc_1$ and $s = \pm ikc_2$. Furthermore, the integrand also possesses a number of poles. Introducing c given by $s = ikc$ (i.e. c is the phase velocity) the poles are determined by

$$D(c, k) \equiv R \cos km_0 h + Q \sin km_0 h = 0 \tag{4.10}$$

$$\bar{\Phi}_{zz}-k^2m_1^2\bar{\Phi}=\frac{(s-ikc_R)(1+\beta_2^2)a(k)}{k\beta_1c_1^2(1-\beta_2^2)}\exp(k\beta_1z)$$

(4.2)
$$\bar{\Psi}_{zz}-k^2m_2^2\bar{\Psi}=\frac{2i(s-ikc_R)a(k)}{kc_2^2(1-\beta_2^2)}\exp(k\beta_2z)$$

$$\bar{X}_{zz}+k^2m_0^2\bar{X}=0$$

where

(4.3)
$$m_1^2=1+s^2/k^2c_1^2,\quad m_2^2=1+s^2/k^2c_2^2,\quad m_0^2=-1-s^2/k^2c_0^2$$

The boundary conditions (2.9), (2.11), (2.12) and (2.13) gives

(4.4)
$$\begin{aligned}
&\bar{X}=0 && z=h\\
&\rho c_2^2\{(c_1^2/c_2^2-2)(\bar{\Phi}_{zz}-k^2\bar{\Phi})+2(\bar{\Phi}_{zz}+ik\bar{\Psi}_z)\}-\rho_0s\bar{X}=0 && z=0\\
&2ik\bar{\Phi}_z-\bar{\Psi}_{zz}-k^2\bar{\Psi}=0 && z=0\\
&s\bar{\Phi}_z+iks\bar{\Psi}-\bar{X}_z=a(k) && z=0
\end{aligned}$$

Applying the boundary conditions, the solutions of (4.2) take the forms

(4.5)
$$\bar{\Phi}=C_1\exp(km_1z)-\frac{(1+\beta_2^2)a(k)}{k\beta_1(1-\beta_2^2)(s+ikc_R)}\exp(k\beta_1z)$$

$$\bar{\Psi}=C_2\exp(km_2z)-\frac{2ia(k)}{k(1-\beta_2^2)(s+ikc_R)}\exp(k\beta_2z)$$

$$\bar{X}=C_3\sin km_0(h-z)$$

where C_1, C_2 and C_3 are given by

(4.6)
$$C_1=-\frac{ic_R(1+m_2^2)a(k)}{sm_1(1-m_2^2)}\frac{Q\sin km_0h}{(s+ikc_R)D}$$

$$C_2=\frac{2c_Ra(k)}{s(1-m_2^2)}\frac{Q\sin km_0h}{(s+ikc_R)D}$$

$$C_3=\frac{ic_Ra(k)R}{m_0(s+ikc_R)D}$$

with

(4.7)
$$R=(1+m_2^2)^2-4m_1m_2$$

$$Q=\rho_0m_1s^4/\rho m_0k^4c_2^4$$

$$D=R\cos km_0h+Q\sin km_0h\,.$$

For $h=5$ km, this means a distance $l=1500$ km, or about 15 degrees of longitude along the equator.

For higher values of n, the damping is still more effective. We therefore conclude that the effect studied above apparently is a very efficient damping mechanism for short period waves, giving results in agreement with observations. It must however, be stressed that the approximation introduced involving no coupling is a very rough one and, for large values of time, must necessarily lead to erroneous results. The most serious objection in this respect is that the approximation above incorporates the assumption that all the energy in the bottom layer is available to be drawn from the bottom layer to the fluid layer. This is, of course, by no means true, as will be demonstrated in the next chapter, where we shall consider the realistic case of weak coupling. Another objection, which does not seem to be really serious, is that we have assumed that the amplitude $a(k)$ is independent of time in the first part of the derivation and later have considered it to vary with time. If the variation with time is relatively slow, this may still be a good approximation.

4. The case of weak coupling. In the case discussed above, we obtained resonance for an infinite set of discrete k-values, leading to an infinite sum of harmonic wave trains located behind the moving corrugation. When the coupling between the two layers are taken into account, this resonance vanishes. If, however, the coupling is weak, we shall almost get resonance, and, as mentioned above, for moderate values of time the cases will be rather equal. For large values of time, however, the effect of the coupling will be dominating. To obtain a problem which may correspond to a seismic wave entering an ocean region, we shall assume that at $t=0$, the uppermost layer (the ocean) is at rest whereas the interface has a form given by (3.1), and the displacements and velocities in the bottom layer are those corresponding to a Rayleigh wave. The initial values of ϕ, ψ, ϕ_t, ψ_t are then found from (7.2) and (7.4), putting $t=0$. Futhermore $\chi=\chi_t=0$ at $t=0$. Thus we initially permit a discontinuity in the velocities at the interface.

Applying Fourier's theorem, we may write

$$
\begin{aligned}
\phi &= \mathrm{Re}\int_0^\infty \Phi(k,z,t)\exp(ikx)dk \\
\psi &= \mathrm{Re}\int_0^\infty \Psi(k,z,t)\exp(ikx)dk \qquad (4.1)\\
\chi &= \mathrm{Re}\int_0^\infty X(k,z,t)\exp(ikx)dk
\end{aligned}
$$

We shall also apply the theory of the Laplace transform. Let $\overline{\Phi}(k,z,s)$ denote the Laplace transform of $\Phi(k,z,t)$. etc. We then obtain from (2.1), (2.3) and (2.4), taking into account the initial conditions, It is noted that the energy transport (3.14) is given in the form of an infinite series, indicating that the energy transport takes place only for discrete values of k. However, the energy store (3.15) is given as a continuous function of k. This is a discrepancy which, of course, is due to the approximations implied by the assumption of no coupling.

Let us first notice that the energy transport takes place for k-values for which $k=k_n=(2n+1)\pi/2\beta_0 h$. The corresponding periods are given by $T_n=2\pi/k_n c_R = 4\beta_0 h/(2n+1)c_R$. The largest period is seen to be $T_0=4\beta_0 h/c_R$. Let us introduce $c_R=4{,}1$ km/sec, $c_0=1{,}5$ km/sec and $h=5$ km, (which is an average value for the Pacific). We then obtain $T_{max}=T_0=12{,}4$ sec. This means that with these typical values of the parameters the energy flux takes place for periods of 12,4 sec and less. In other words, energy corresponding to periods of 12,4 sec and less are drawn from the bottom layer and conveyed to the fluid layer (the ocean) and thereby finally lost for the seismic wave system. This result is in good agreement with observations (see Ewing, Jardetzky and Press (1957) pp. 172—174) where it is pointed out that the energy for periods less than about 12 sec is absent for waves which have propagated longer distances across ocean basins.

To get an idea of the order of magnitude of the damping mechanism, we replace the integral in (3.15) by a sum of the same type as the right hand side of (3.14). This means that we assume that the energy flux taking place at wave number $k=k_n$ takes its energy from a k-interval of width $\pi/\beta_0 h$. Furthermore, we assume that $a(k)$ is a function of time, varying relatively slowly, such that it suffices to take its variation with time into account only from now on. We then have, applying (3.16) and (3.14)

(3.17)
$$\frac{d}{dt}\left(\frac{E(k_n,t)\pi}{\beta_0 h}\right)=-\frac{\rho_0\pi^2 c_R^3}{\beta_0^2 h}k_n^2 a(k_n,t)^2$$

or

(3.18)
$$\dot{a}(k_n,t)/a(k_n,t)=-\alpha_n c_R/h$$

with $\alpha_n=\rho_0 k_n h/2\rho B\beta_0$.

Hence

(3.19)
$$a(k_n,t)/a(k_n,0)=\exp(-\alpha_n c_R t/h)\,.$$

Let us introduce $n=o$, $\rho=3\rho_0$, $\nu=0{,}25$, $c_R=4{,}1$ km/sec, $c_0=1{,}5$ km/sec, which gives $\alpha_0=0{,}01$. Let l be the distance the wave has advanced, i.e. $l=c_R t$. Then

$$a(k_0,l)/a(k_0,0)=\exp(-0{,}01l/h) \quad \text{or}$$

$$E(k_0,l)/E(k_0,0)=\exp(-0{,}02l/h)\,.$$

Let us for example put $l=300h$ which gives

$$E(k_0,l)/E(k_0,0)=e^{-6}\approx 2{,}5\,10^{-3}\,.$$

Therefore, when the wave has advanced a distance $l=300h$, the energy (for wave periods less than about 12,4 sec) is reduced to about 0,25 per cent of the initial value.

Cauchy's theorem then leads to

$$p(o)=\rho_0 c_R^2/\beta_0 \mathrm{Im} \int_0^\infty \frac{ka(-ik)\sin hk\beta_0 h}{\cos hk\beta_0 h}\exp(k(x-c_R t))dk + 2\rho_0\pi c_R^2/\beta_0^2 h \sum_{n=0}^{\infty} k_n a(k_n)\sin k_n(x-c_R t) \tag{3.10}$$

when $x-c_R t<0$ and

$$p(o)=\rho_0 c_R^2/\beta_0 \mathrm{Im} \int_0^\infty \frac{ka(-ik)\sin hk\beta_0 h}{\cos hk\beta_0 h}\exp(-k(x-c_R t))dk \tag{3.11}$$

when $x-c_R t>0$.

It is noted that the pressure is non-symmetrical (in spite of the symmetrical form of the moving corrugation), there being an infinite number of harmonic wave trains behind the corrugation. That the wave trains are located *behind* the corrugation also follows from the fact that the group velocity is less than the phase velocity. This is readily seen to be true from the expression for the phase velocity, c, for the free waves in the fluid layer which is easily found to be given by

$$c=c_0\{1+((2n+1)\pi/2kh)^2\}^{\frac{1}{2}} \tag{3.12}$$

from which it follows that

$$cc_g=c_0^2 \tag{3.13}$$

where c_g denotes the group velocity.

We shall be interested in finding the upward flux of energy per unit time at the interface. This is found from (3.8), (3.10) and (3.11) to be given by

$$\int_{-\infty}^{\infty} p(o)w(o)dx=\rho_0\pi^2 c_R^3/\beta_0^2 h \sum_{n=0}^{\infty} k_n^2 a(k_n)^2. \tag{3.14}$$

The important question now is: What is the order of magnitude of this energy transport compared to the energy store in the bottom layer? As pointed out above, the case of no coupling, strictly speaking, corresponds to $\rho_0/\rho\to 0$. To get an idea of the order of magnitude, we shall apply the formulas for realistic values of the densities, and thus we put $\rho_0/\rho=1/3$. Let ε denote the total energy in the Rayleigh wave, and $E(k)$ the contribution to ε from waves with wave number k such that

$$\varepsilon=\int_0^\infty E(k)dk. \tag{3.15}$$

We then have (see Appendix, section 7).

$$E(k)=\rho\pi c_R^2 Bka(k)^2 \tag{3.16}$$

where B denotes a coefficient depending only on the Poisson ratio ν. For $\nu=0{,}25$, $B=\sqrt{6\sqrt{3}}\approx 3{,}224$.

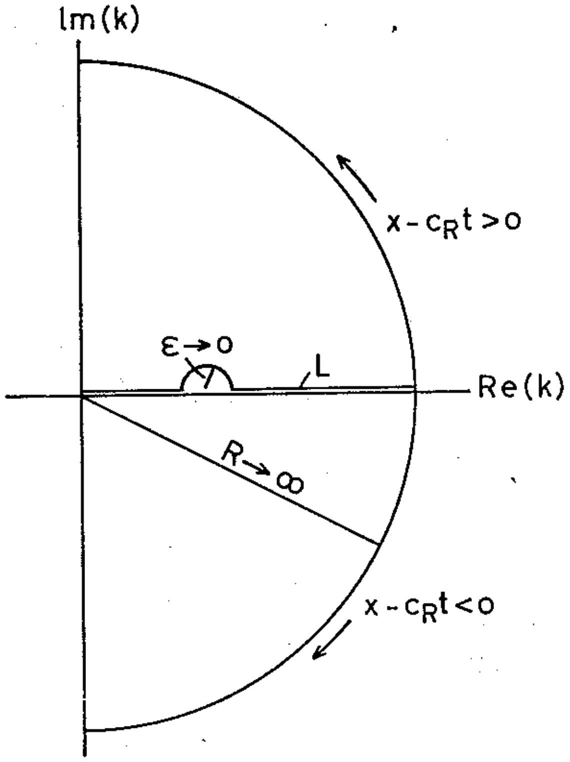

Figure 1. The path of integration.

With X given by (3.5), the pressure p and the vertical velocity, w, take, at $z=0$, the forms

$$p(o) = -\rho_0 c_R^2/\beta_0 \mathrm{Re} \int_0^\infty \frac{ka(k)\sin k\beta_0 h}{\cos k\beta_0 h} \exp(ik(x-c_R t))dk \tag{3.7}$$

$$w(o) = -c_R \mathrm{Re} \int_0^\infty ika(k)\exp(ik(x-c_R t))dk. \tag{3.8}$$

It is to be noted that the integral in (3.7) is an improper integral, the integrand being infinite for the values of k corresponding to $\cos k\beta_0 h = 0$ (i.e. $k = k_n = (2n+1)\pi/2\beta_0 h$ where n denotes an arbitrary integer.) Physically this is due to the fact that the problem has been treated as a stationary one. If the problem had been attacked as an initial value problem, the corresponding integral would have been a proper one such that, as $t \to \infty$, (3.7) would have been replaced by (compare for example Palm (1953))

$$p(o) = -\rho_0 c_R^2/\beta_0 \mathrm{Re} \int_L \frac{ka(k)\sin k\beta_0 h}{\cos k\beta_0 h} \exp(ik(x-c_R t))dk \tag{3.9}$$

where the path of integration, L, is shown in Fig. 1. For simplicity, we consider only symmetrical elevations of the interface such that $a(k)$ is real for real values of k. When the path of integration is deformed as suggested in the figure, application of

At the interface $(z=0)$ the dynamic conditions require

(2.10) $$\sigma_z=-p, \qquad \tau_{zx}=0 \qquad z=0$$

or, by means of (2.4), (2.5) and (2.6),

(2.11) $$\rho c_2^2\{(c_1^2/c_2^2-2)(\phi_{xx}+\phi_{zz})+2(\phi_{zz}+\psi_{xz})\}-\rho_0\chi_t=0 \qquad z=0$$

(2.12) $$2\phi_{xz}+\psi_{xx}-\psi_{zz}=0 \qquad z=0$$

Here ρ_0 denotes the density in the fluid layer. Furthermore, the requirement of continuous normal velocity at the interface gives

(2.13) $$\phi_{zt}+\psi_{xt}-\chi_z=0 \qquad z=0$$

3. The case of no coupling. We now assume that the motion in the bottom layer is independent of the occurrence of the uppermost layer. This assumption corresponds to $\rho_0/\rho\to 0$. Furthermore, only the Rayleigh phase of the motion is taken into account. The elevation, ζ_0, of the interface may then be written

(3.1) $$\zeta_0=\mathrm{Re}\int_0^\infty a(k)\exp(ik(x-c_Rt))dk$$

where c_R denotes the Rayleigh velocity, k the wave number and $a(k)$ an arbitrary, known, function of k. The boundary conditions take the form

(3.2) $$\begin{aligned}\chi_t&=0 \qquad & z&=h\\ \chi_z&=\zeta_{0t} \qquad & z&=0\end{aligned}$$

After some time the motion of the fluid will be stationary in a frame of reference fixed to the corrugation of the interface. The velocity potential may then be written

(3.3) $$\chi=\mathrm{Re}\int_0^\infty X(z,k)\exp(ik(x-c_Rt))dk.$$

From (2.1) we find

(3.4) $$X_{zz}+k^2(c_R^2/c_0^2-1)X=0.$$

The solution of this equation satisfying (3.2) is

(3.5) $$X=\frac{ic_Ra(k)\sin k\beta_0(h-z)}{\beta_0\cos k\beta_0 h}$$

where β_0 is defined by

(3.6) $$\beta_0^2=c_R^2/c_0^2-1.$$

2. Equations of motion and boundary conditions. As mentioned above, our system consists of two layers, the uppermost layer being a perfect homogeneous fluid and the bottom-layer being a perfect elastic medium. We introduce a frame of references with the z-axis pointing vertically upwards and the x-axis coinciding with the interface of the two layers. All the equations are linearized. Neglecting the effect of gravity, the equation governing the motion in the fluid layer then takes the form

$$\chi_{xx}+\chi_{zz}-c_0^{-2}\chi_{tt}=0 \; * \tag{2.1}$$

where χ is the velocity potential and c_0 the (constant) velocity of sound. In the elastic bottom layer we introduce the displacement vector (ξ, ζ) and the displacement potentials ϕ and ψ such that

$$\xi=\phi_x-\psi_z \qquad \zeta=\phi_z+\psi_x \tag{2.2}$$

We then have

$$\phi_{xx}+\phi_{zz}-c_1^{-2}\phi_{tt}=0 \tag{2.3}$$

$$\psi_{xx}+\psi_{zz}-c_2^{-2}\psi_{tt}=0 \tag{2.4}$$

where c_1 denotes the dilatational velocity given by

$$\rho c_1^2=\lambda+2\mu \tag{2.5}$$

and c_2 denotes the distortional velocity given by

$$\rho c_2^2=\mu \tag{2.6}$$

Here ρ is the density in the bottom layer and λ and μ are the Lamé constants.

Let the normal stress and shearing stress acting on an element oriented normal to the z-axis be denoted by σ_z and τ_{zx}, respectively. We then have

$$\begin{aligned} \sigma_z&=\lambda(\phi_{xx}+\phi_{zz})+2\mu(\phi_{zz}+\psi_{xz}) \\ \tau_{zx}&=\mu(2\phi_{xz}+\psi_{xx}-\psi_{zz})\,. \end{aligned} \tag{2.7}$$

The boundary conditions at the free surface of the fluid layer may be written

$$p=0 \qquad z=h \tag{2.8}$$

where p denotes the pressure and h the depth of the fluid layer. (2.8) may also be written

$$\chi_t=0 \qquad z=h \tag{2.9}$$

* It should be noted that subscripts x, z and t denote differentiation, except in connection with the stresses where σ_z and τ_{zx} denote the components of the stress tensor.

To get some insight in the problem we will, as a first rough approximation, assume that the motion in the bottom layer is not influenced by the occurrence of the uppermost layer. The motion in the uppermost layer is then due to a (known) travelling corrugation of the bottom, and the problem is rather easily solved. We will call this case the case of no coupling, in contrast to the more realistic problem in which the two layers are considered as one (coupled) system which will be treated later in the paper. It will turn out that the degree of coupling depends on the ratio of the density in the two layers such that a small density in the uppermost layer compared to the density in the lowest layer corresponds to weak coupling. Furthermore, we consider only the Rayleigh phase of the motion in the bottom layer, such that the bottom corrugation travels with a constant velocity (the Rayleigh velocity) and without change of form.

The motion which is set up by such a moving corrugation of the bottom is in principle well known. Let us for the moment assume that the fluid is incompressible. The moving corrugation will then create a system of gravity waves (surface waves) in the case of $U<(gh)^{\frac{1}{2}}$ where U denotes the velocity of the corrugation, g the acceleration of gravity and h the depth of the fluid. These waves may be described as an infinitely long harmonic wave train located *behind* the corrugation. Since the wave motion is asymmetric with respect to the corrugation (even if this is symmetric), there will be a net pressure force acting on the corrugation such that energy is continuously conveyed to the fluid. The fluid will therefore, as time increases, receive an increasing amount of energy. A Fourier analysis of the energy will reveal that almost all the energy has a definite period (corresponding to that of the infinitely long wave-train). This energy is transferred to the fluid from the bottom layer. In the case of no coupling, however, the bottom layer has an infinite store of energy, and therefore the loss of energy does not lead to any changes in this layer.

When $U>(gh)^{\frac{1}{2}}$, no surface waves will occur. Therefore the net pressure force acting on the corrugation is zero, and no such energy transfer occurs.

The Rayleigh velocity is much larger than $(gh)^{\frac{1}{2}}$, and the gravity waves play no important role in the present problem. Therefore, we shall henceforth neglect the effect of gravity. On the other side, the Rayleigh velocity is also larger than the velocity of sound for the fluid. Thus the corrugation moves with a *supercritical* velocity. It is, however, well known that a body moving with a supercritical velocity will initiate a wave train behind the body, and thereby be exposed to a net pressure resistance of very much the same type as that due to gravity waves discussed above. Taking into account the compressibility of the fluid we shall therefore find that in the case of no coupling, energy is continuously conveyed to the fluid. Since in this case (in contrast to the case of gravity waves) we shall find an *infinite* number of harmonic wave trains, almost all the energy will be located on an infinite set of discrete periods.

In the more realistic case, where the coupling is taken into account, the energy store in the bottom layer is finite. The mathematical problem thereby becomes more complicated, but, as we shall see, definite conclusions about the energy transfer may be drawn also in this case.

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THE ENERGY TRANSFER FROM SUBMARINE SEISMIC WAVES TO THE OCEAN

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Summary. Seismic waves travelling across ocean basins, will lose energy to the ocean. This energy transport is irreversible and therefore represents a kind of dissipation. In the present paper this energy transport is computed. It is found that for an ocean depth characteristic for the Pacific, only very little energy of period larger than about 11,8 sec is lost to the ocean, whereas a seismic wave loses up to half of its initial energy of period less than 11,8 sec. Therefore, by a spectral analysis of seismic waves which have travelled across an ocean basin, one should expect short period waves (less than about 11,8 sec) to have relatively less energy than longer period waves. Therefore, spectral analysis of seismic waves which have travelled across an ocean basin and seismic waves which have travelled across a continent should show a marked difference in character for short period waves (periods less than about 11,8 sec).

1. Introduction. A seismic wave which starts on a propagation across ocean basins, will set the ocean in a state of wave motion. The energy thus conveyed to the ocean is taken from the wave energy in layers below the ocean. When the seismic wave afterwards starts on a propagation across the continents, the energy which has been conveyed to the ocean, is lost for the seismic wave. This energy transfer therefore constitute a kind of damping mechanism. It is our intention to examine in the present paper this energy transfer, its order of magnitude and for which wave lengths (periods) the transfer has its maximum intensity. A study of these problems obviously may throw some light on the puzzling observational fact that short period seismic waves in typical ocean areas suffer a much greater attenuation than longer period waves (see for example Ewing, Jardetzky and Press (1957)).

We will in this paper consider only the two-dimensional case. The real system consisting of the ocean and the layers below will be replaced by a two-layer model in which the uppermost layer (the ocean) is a perfect fluid of uniform depth and the lowest layer is a homogeneous and isotropic elastic-solid bottom of infinite vertical extent. Both layers are of infinite horizontal extent, and there is no friction.

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Vol. XXVI.

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