A STUDY OF THE LIMITING POLARIZATION OF HIGH FREQUENCY RADIO WAVES REFLECTED VERTICALLY FROM THE IONOSPHERE

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(Manuscript received 20th January 1955)

Abstract: The ray treatment of the magneto-ionic theory shows that high frequency radio waves, reflected vertically at the ionosphere, should be very nearly circularly polarized for the geomagnetic conditions at Kjeller, $\varphi = 59° 58'$ N, $\lambda = 11° 6'$ E. Gv. (angle θ between the direction of propagation and the direction of the terrestrial magnetic field equals 17°), and at Tromsø, $\varphi = 69°$ $59'$ N, $\lambda = 18° 46'$ E. Gr. ($\theta = 12°$).

Considering the propagation in a horizontally stratified ionosphere, it is found that according to a full wave theory (including the effects of magneto-ionic coupling), the reflected waves should still be nearly circularly polarized.

Possible effects on the limiting polarization of horizontal gradients and irregularities are discussed, and a statistical theory of limiting polarization has been developed in which the reflected wave has been treated as a sum of rays reflected within a cone about the vertical direction, the phases of the individual rays being distributed at random. It is concluded that this mechanism may explain departures from circularity of the limiting polarization. The effect of a specularly reflected component in addition to a continuous distribution of rays is also treated.

Polarization measurements have been made both at Kjeller and at Tromsø and the experimental equipment used is described. Twenty observations of the sense of polarization (sense of rotation of the field vectors) of the magneto-ionic third component, the z-component, all showed that this component is polarized in the same way as the ordinary magneto-ionic component. This is in accordance with present theories which explain the z-component by a coupling mechanism.

During quiet conditions, the reflected waves both at Kjeller and at Tromsø, were found to be in general nearly circularly polarized. However departures from circularity were observed, and these may be explained by the above statistical theory of limiting polarization. It was also found in accordance with this theory, that the probability of observing departures from circularity is greater during conditions of fading minima than during conditions of fading maxima. Furthermore it was found, also in accordance with the statistical theory, that the probability of observing departures from circularity was greater during conditions when the observed amplitude distribution was typically «Rayleigh» than during conditions when the amplitude distribution was rather «displaced Gaussian» than «Rayleigh».

Echoes from layers which are formed at the E-layer levels during polar geomagnetic storms, were found to be in general more elliptically polarized, and the major axis of the polarization ellipse for these echoes was nearly always orientated in East-West direction. This result may be explained by the statistical theory if one assumes a nonsymetrical model of the ionosphere. The result may also be explained using the full wave theory, by making certain assumptions about the electron concentration in the lower ionosphere during disturbed conditions.

1.0 INTRODUCTION AND OUTLINE

The three fundamental parameters observed during radio echo measurements are:

a) The virtual height of the reflecting layers, usually measured as a function of frequency,

b) The field strength of the reflected waves,

c) The state of polarization of the reflected waves.

Determination of the first two of these parameters gives the electron density and reflecting properties of the layers, information which is required for the calculation of the MUF (Maximum Usable Frequency) and the LUHF (Lowest Usable High Frequency). Knowledge of the state of polarization is of both practical and theoretical interest. It is closely linked with the practical problem of «night error» in direction finding, and has theoretical interest in connection with fading and more fundamental studies of magneto-ionic coupling processes, i. e. coupling between the ordinary and extraordinary component waves. Polarization measurements have also provided a crucial check of a number of theoretical points in connection with the magneto-ionic theory of radio wave propagation as first given by Appleton (1).

It is well known that according to a simple ray theory, the limiting polarization of radio waves emerging from the ionosphere is determined by the characteristics at the lower edge of the ionosphere. Since Appleton and Ratcliffe (2) in 1928 first demonstrated experimentally the elliptical polarization of reflected radio waves, a number of investigations have been carried out showing qualitative agreement with the simple ray theory, but certain effects have been observed which cannot be explained by this theory. All observers agree that the polarization ellipse changes continuously both in size and shape, and these changes seem difficult to explain as being due to changes of the characteristics at the lower edge of the ionosphere. White and Ratcliffe (3) found in 1933 a correlation between the field strength and polarization of reflected waves; strong signals usually giving an approximation to circular polarization and weak signals giving a more elliptic polarization. This result is in contradiction with the ray theory because absorption here would tend to make reflected waves more circularly than elliptically polarized.

Recently polarization measurements have been made by Morgan (4), showing a marked variation of the limiting polarization of short waves with frequency. As the test frequency approached the critical frequency (the frequency where there is a marked group retardation), the polarization of the echoes became nearly linear.

In section (2) of this communication the magneto-ionic theory of Appleton will be outlined, and the limitations of the theory discussed.

The problem of determining the conditions when the ray theory may be used is closely linked with the problem of magneto-ionic coupling. In section (3) the effects of magneto-ionic coupling on the limiting polarization of the reflected waves will be treated. A theory which includes the coupling is called a wave theory of limiting polarization. The wave theoretical treatment of Budden (5) is included in section (3).

In section (4) a statistical theory of limiting polarization of reflected waves will be developed. In this theory the reflected wave is treated as a sum of rays reflected within a cone about the vertical direction, the phases of the individual rays being treated as random. The effect of a specularly reflected component in addition to the continuous distribution of rays is also treated.

At the Norwegian Defence Research Establishment at Kjeller, a series of polarization measurements was carried out during October and November 1950, and at the Auroral Observatory at Tromsø during September and October 1951, the object of the measurements being as follows:

a) To determine the sense of polarization of the *third* magneto-ionic component, the *z*-component,

b) To study the variations of the limiting polarization at Kjeller and at Tromsø,

c) To study in detail, echoes from layers which are formed at the E-layer levels during terrestrial magnetic storms at Tromsø.

The experimental technique used during these measurements was the standard system using crossed loops, separate receiving channels and an oscillographic representation of the polarization ellipse. In 1953 the radio polarimeter was modified so that two parameters determining the state of polarization of the reflected pulses could be recorded continuously by a recording milliammeter. Using this technique a new series of measurements was made at Tromsø during September and October 1953, and at Kjeller during May 1954.

In section (5) the experimental equipment will be described, and in section (6), the experimental determination of the sense of polarization of the z-component will be given. Observations of limiting polarization are given in section (7), and these are discussed in section (8). In section (9) the complete results are summarized.

2.0 THEORY OF RADIO-WAVE PROPAGATION IN A HORIZONTALLY STRATIFIED IONOSPHERE

The magneto-ionic theory is the theory of radio wave propagation in an ionized medium, in the presence of an external magnetic field. As the magnetic field renders the medium anisotropic, the theory is complex, and the full wave solution has not yet been given. We start with the wave equation

$$\nabla^2\vec{E} - \nabla\nabla\ .\ \vec{E} - \varepsilon_o\mu_o\dot{\vec{E}} - \mu_o\ddot{\vec{P}} = 0, \qquad (2.1)$$

which follows in the usual way from the Maxwell equations. The current density $\vec{J}$ has been put equal to zero, since the currents due to vibrating ions have been treated as polarization currents and not as conduction currents. The electric polarization $\vec{P}$ is then defined as the volume density of electric dipole moments, $e\vec{r}$, where e is the ion charge and $\vec{r}$ the ion displacement.

In addition to equation (2.1) we need an equation

$$\vec{P} = \varepsilon_o\,\chi\,\vec{E}, \qquad (2.2)$$

giving the properties of the medium. Equation (2.2) is identical with the equation of motion of the ions. The electric susceptibility χ, can therefore be determined from this equation by inspection.

2.1 Isotropic Medium.

We assume variations with time as $exp\ (j\omega t)$, and take the x_3-axis as being the axis of phase propagation. Only plane waves incident vertically on a horizontally stratified medium are to be considered, and the effect of heavy ions is neglected. Equation (2.1) then reduces to

$$\frac{d^2E}{dx_3^2} + k^2E = 0, \qquad (2.3)$$

where,

$$k^2 = \frac{\omega^2}{c^2}\,(1+\chi) = \frac{\omega^2}{c^2}\left(1 - \frac{jNe^2}{(\nu+j\omega)\,\omega m\varepsilon_o}\right), \qquad (2.4)$$

and where N is number of free electrons per unit volume, ν the mean collisional frequency, e the charge of the electron and m the mass of the electron. The value used for χ follows from section (2.2).

For a homogeneous medium k is a constant, and the solution of equation (2.3) is then,

$$E = A\ exp(jkx_3) + B exp(-jkx_3), \qquad (2.5)$$

giving the foreward and backward propagated waves. If the medium is not homogeneous, we may try the solution

$$E = A(x_3)\ exp(j\int kdx_3) + B(x_3)\ exp(-j\int kdx_3). \qquad (2.6)$$

where A and B are functions of x_3 to be determined. It is found by inspection that both functions on the right hand side of equation (2.6) satisfy the differential equation (2.3), if A is proportional to $1/\sqrt{k}$ and,

$$\left(\frac{dn}{dx_3}\,\frac{\lambda}{2\pi n}\right)^2 \ll 1,$$
$$\frac{1}{n}\,\frac{d^2n}{dx_3^2}\left(\frac{\lambda}{2\pi}\right)^2 \ll 1. \qquad (2.7)$$

Equations (2.7) formulate the conditions determining the validity of the simple ray theory. The solutions (2.6) in this case will be the well known W.K B.-approximations.

Equations (2.7) show that for an ionospheric layer having no abrupt changes in the electron density distribution, the ray theory may be used except where $n \approx 0$. If $n \approx 0$ there will be a strong coupling between the two components of the propagated waves. In those parts of the ionosphere where deviation occurs we have no reason to believe that the simple ray theory can be used. In order to

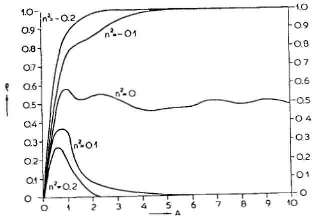

Fig. (2.1). Reflection coefficient ϱ for a parabolic layer as function of the layer thickness A in vacuum wavelengths, for parameter values of n^2_{min}.

investigate the propagation in such regions more closely, we have in Fig. (2.1) drawn curves which give the reflection coefficient ϱ for typical ionospheric layer models.[1] The electron density distribution is parabolic. As abscissa is chosen the layer thickness in vacuum wavelengths, and the curve parameters are the maximum electron density of the layers (represented by the minimum value of n^2). The method used in the calculations is the one given by Hartree (6), and an outline of the technique is given in Appendix 1 at the end of this paper.

The conclusion drawn from Fig. (2.1) is that the reflection coefficient is determined by the critical electron density, (the electron density that would give reflection by ray theory), except for very thin layers. A number of investigations of this type have been made by different workers, and have all given the same results. These investigations show also that the results hold for different layer models.

2.2 Anisotropic Homogeneous Medium.

Due to the terrestrial magnetic field an electromagnetic wave which is propagated through the ionosphere gives rise to polarization currents in the direction of phase propagation. $\nabla \cdot \vec{E}$ will therefore not be zero and the electric susceptibility χ is represented by a tensor of second order. We shall consider the case of a homogeneous medium and may assume therefore a variation with the coordinate in the direction of phase propagation x_3 as $\exp(-jkx_3)$. Equations (2.1) and (2.2) will then together form a system of three homogeneous equations in the three components of the electric polarization $\vec{P}$. The secular relation for this system determines k as a function of the parameters of the medium. The secular relation is,

$$\frac{k^2c^2}{\omega^2} = \left(n - jc\frac{\varkappa}{\omega}\right)^2 = 1 - \frac{X}{1 - jz - \dfrac{y_T^2}{2(1-jz-x)} \pm \sqrt{\left(\dfrac{y_T^2}{2(1-jz-x)}\right)^2 + y_L^2}} \tag{2.8}$$

In equation (2.8) the following symbols have been introduced:

$$x = \frac{Ne^2}{\varepsilon_0 m\omega^2}$$

$$\vec{y} = \frac{\vec{\omega}_M}{\omega} = \frac{\vec{B}_0 e}{m\omega}$$

$$z = \frac{\nu}{\omega}$$

$y_{T,L} = y\sin\theta,\ y\cos\theta$, where θ is angle between the terrestrial magnetic field $\vec{B}_0$ and the direction of phase propagation.

We have here considered only the propagation of plane waves for which the ratio E_2/E_1 must be constant. From equations (2.1) and (2.2) we obtain:

$$Q = \frac{E_2}{E_1} = \frac{P_2}{P_1} = \frac{D_2}{D_1} = -\frac{H_1}{H_2} = -\frac{B_1}{B_2} \tag{2.9}$$

$$= j\left[\frac{y_T^2}{2y_L(1-jz-x)} \pm \sqrt{\left(\frac{y_T^2}{2y_L(1-jz-x)}\right)^2 + 1}\right]$$

The axis system has been chosen so that the terrestrial magnetic field lies in the x_2-plane.

Equation (2.9) determines the complex polarization of a plane wave which is propagated through the medium. The equation shows that only two characteristic waves can occur, and for each of these the refractive index n and the absorption index $\varkappa$ are given by equation (2.8). Equations (2.8) and (2.9) are usually called the Appleton-Hartree equations.

From equation (2.9) it follows that the polarization of the two characteristic waves which can be

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[1] The numerical calculations have been made by cand. real. J. Garwick of the Mathematical Section of the NDRE.

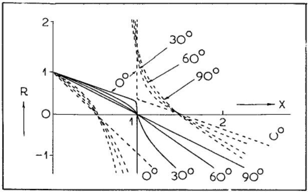

Fig (2.2) Dispersion curves for $y = 0.5$ and parameter values of the angle θ assuming zero collisions.

$$R = \left(n - jc\frac{\varkappa}{\omega}\right)^2$$

propagated in the medium are represented by inverse complex quantities. These characteristic waves are elliptically polarized and have different senses of rotation of the field vectors.

2.3 Anisotropic Inhomogeneous Medium, Ray Theory.

If the medium is not homogeneous and use is made of a ray theory, the propagation in a horizontally stratified ionosphere at vertical incidence may still be described by equations (2.8) and (2.9). In Fig. (2.2) is plotted $\left(n - jc\frac{\varkappa}{\omega}\right)^2$ from equation (2.8) as function of x for different values of θ for a fixed frequency $f = 2f_H$, assuming zero collisions.

Fig. (2.2) represents the propagation conditions in the ionosphere. x is proportional to the electron density N, positive values of n applying to waves propagated upwards and negative values of n applying to waves propagated downwards. There are three levels where n becomes zero, at $x = 1 - y$, $x = 1$ and $x = 1 + y$. These levels correspond to reflection levels. The wave which is reflected at $x = 1$ will not be influenced by the terrestrial magnetic field in the case of transverse propagation and has therefore been called the *ordinary* magneto-ionic component. The wave which is reflected at $x = 1 - y$ has been called the *extraordinary* magneto-ionic component. The branch for which x becomes zero at $x = 1 + y$ has no application in the ray theory because this branch is isolated from energy from below.

The solution of equation (2.9) may be taken from a nomogram in the case of zero collisions. This is shown in Appendix 2, and the nomogram is given in Fig. (2.3).

The nomogram shows quite clearly the different polarization of the waves. For longitudinal propagation ($\theta = 0$), the waves will be circularly polarized for all values of x and y, while transverse propagation gives linear polarization for all values of x and y. At $x = 1$ the waves are linearly polarized, and for very high frequencies ($y \to 0$) the waves will be circularly polarized for all values of θ and x.

If collisions are taken into account, then the solution of equation (2.9) may still be taken from a nomogram. This has been done by Bailey and is

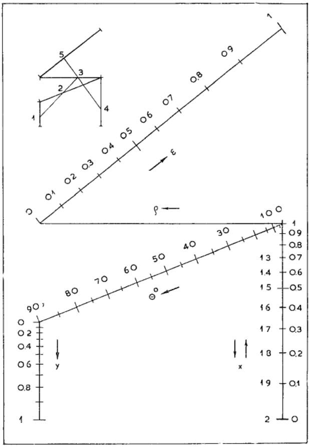

Fig. (2 3) Nomographic representation of the polarization assuming zero collisions. For the ordinary magneto-ionic component the major axis ε of the ellipse will be in geomagnetic NS-direction and for the extraordinary component the major axis will be in geomagnetic EW-direction.

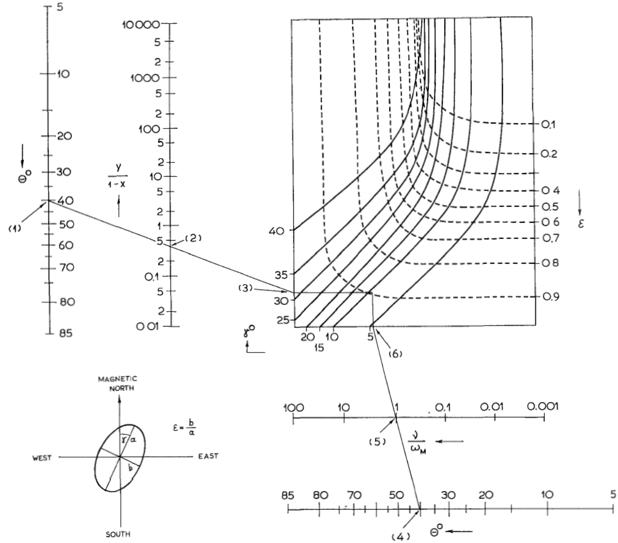

Fig. (2.4). Nomographic representation of the state of polariaztion of the magneto-ionic components. ε is the ratio of the axes of the polarization ellipse and γ gives the orientation of the ellipse For the ordinary magneto-ionic component the major axis is rotated from NS-direction in the zero collision case towards NE-SW-direction and the extraordinary ellipse is rotated from EW-direction towards NE-SW-direction.

also described by Scott (7). In Fig. (2.4) a modified form of Baileys nomogram is given.

The nomogram illustrates the way in which large values of ν will make the waves nearly circularly polarized even for large values of θ. At the lower edge of the ionosphere ν will be considerable, and we may therefore deduce that radio waves which are reflected from the ionosphere, according to the simple ray theory, will be nearly circularly polarized even at moderately high geomagnetic latitudes.

In Fig. (2.2) curves were given for $R = \left(n - jc\frac{\varkappa}{\omega}\right)^2$ as a function of x for different values of θ and a fixed frequency, assuming zero collisions. Dispersion curves where ν is taken into account have been calculated by Taylor (8), Goubau (9) and others, but none of these curves seems to be representative for high geomagnetic latitudes. A set of curves has therefore been drawn at the Norwegian Defence Research Establishment, Landmark and Lied (10)[1], for the magnetic conditions at Tromsø where $\theta = 12°$. In Fig. (2.5) curves are given for a fixed frequency ($y = 0{,}5$). In these curves n and

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